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A Stable Proton without R Parity — Implications for the LSP —

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Abstract

The proton decays too rapidly in supersymmetric theories if a dimension-4 operator $\bar{\mathbf{5}} \cdot \mathbf{10} \cdot \bar{\mathbf{5}}$ exists in the superpotential. The conventional idea is to impose the R-parity to kill this operator with a stable lightest supersymmetry particle (LSP) as a direct consequence. However, the SUSY-zero mechanism is also able to kill the operator without an unbroken R-parity. In this article, we provide a firm theoretical justification for the absence of the dimension-4 proton decay operator under the SUSY-zero mechanism, by using some input from string theory. The LSP may be unstable without the R-parity and, indeed, some dimension-5 R-parity violating operators may be generated in effective theories. This suggests that the dark matter is an axion in this string theory inspired model. An insight on the SUSY-zero mechanism is also obtained.

The $SU(5)_{GUT}$ gauge coupling unification of supersymmetric extensions of the standard model is quite remarkable. Supersymmetric theories, however, allow dimension-4 operators that break baryon number and lepton number, and lead to too rapid proton decay.

The dimension-4 proton decay operators

$$W \ni \bar{D} \cdot \bar{U} \cdot \bar{D} + \bar{D} \cdot Q \cdot L + L \cdot \bar{E} \cdot L \tag{1}$$

are simply written as

$$W \ni \bar{\mathbf{5}} \cdot \mathbf{10} \cdot \bar{\mathbf{5}} \tag{2}$$

in terms of Georgi–Glashow $SU(5)_{GUT}$ unified multiplets. Conventional idea has been to impose a matter parity or R parity to kill these operators. Chiral multiplets $\mathbf{10}$ and $\bar{\mathbf{5}}$ are odd and $H(\mathbf{5})$ and $\bar{H}(\bar{\mathbf{5}})$ are even under the matter parity. The dimension-4 proton decay operator (2) is odd under the parity and vanishes in a theory with the matter parity. Since the R parity is just a combination of the matter parity and $(-1)^F$, where F is the Fermion number, the R parity is equivalent to the matter parity. The lightest supersymmetry particle (LSP) is stable in a theory with an unbroken $\mathbb{Z}_2$ R (and matter) parity [1].

The dimension-4 operator (2) is absent due to the $\mathbb{Z}_2$ matter parity but this argument cannot be turned around and the absence of the operator (2) is not enough to conclude that there is a $\mathbb{Z}_2$ symmetry and that the LSP is stable. Indeed, the operator (2) is absent in the framework proposed in [2], although the $\mathbb{Z}_2$ matter parity is broken. In this article, we rederive the absence of (2) with a plain D = 4 field theory language. We further study the phenomenological consequences of the absence of the R parity, with the LSP decay a particular case. An insight on the SUSY-zero mechanism is also obtained as a bi-product.

The dimension-4 proton decay operator (2) is absent in [2] essentially because of the SUSY-zero mechanism. Let us consider two $SU(5)_{GUT} \times U(1)$ gauge theories with U(1) charges of the chiral multiplets given in Table 1. Those D = 4 field-theory models are simplified versions of models in [2]; ingredients essential to the absence of (2) are not lost in the simplification.[1] Chiral multiplets $\bar{N}^c$ are in the Hermitian conjugate representation of the right-handed neutrino chiral multiplets $\bar{N}$. The U(1) gauge symmetry is not necessarily free of anomaly; the anomaly is cancelled by the generalized Green–Schwarz mechanism. The $\mathbb{Z}_5$ subgroup of the U(1) gauge symmetry of both models is equivalent to the $\mathbb{Z}_5$ centre of the $SU(5)_{GUT}$ symmetry, and $\mathbb{Z}_{10}$ subgroup of the U(1) symmetries gives the matter parity $\mathbb{Z}_{10}/\mathbb{Z}_5 \simeq \mathbb{Z}_2$. In the 4+1 model, the dimension-4 proton decay operator (2) is absent even if

[1] The $SU(5)_{GUT} \times U(1)$ gauge group is embedded in $G = E_7$ or E_8 in string theory in [2]. See the appendix for more about the relation between the models in [2] and those presented in this article.

Chiral multiplets		$\mathbf{10}$	$\bar{\mathbf{5}}$	$H(\mathbf{5})$	$\bar{H}(\bar{\mathbf{5}})$	$\bar{N}$	$\bar{N}^c$
U(1) Charge	4+1 model	-1	3	2	-2	-5	5
U(1) Charge	3+2 model	-3	-1	6	4	-5	5

Table 1: U(1)-charge assignments of (the simplified version of) the two models of [2]. These charge assignments allow all of Dirac neutrino Yukawa couplings $W \ni \bar{N} \cdot \bar{\mathbf{5}} \cdot H(\mathbf{5})$, up-type, down-type and charged-lepton Yukawa couplings. Chiral multiplets with ^{c} are in the Hermitian conjugate representation of those without ^{c} under the $\mathrm{SU}(5)_{\mathrm{GUT}} \times \mathrm{U}(1)$.

chiral multiplets $\bar{N}^c$ have non-zero expectation values, because any operators of the form

$$W \not\ni \bar{\mathbf{5}} \cdot \mathbf{10} \cdot \bar{\mathbf{5}} \cdot \left\langle \bar{N}^c \right\rangle^{n \geq 0} \tag{3}$$

are forbidden by the U(1) gauge symmetry. Likewise in the 3+2 model, some of chiral multiplets $\bar{N}$ may have non-zero expectation values, yet (2) is absent, because

$$W \not\ni \bar{\mathbf{5}} \cdot \mathbf{10} \cdot \bar{\mathbf{5}} \cdot \left\langle \bar{N} \right\rangle^{n \geq 0} \tag{4}$$

are not allowed by the U(1) gauge symmetry. This is so-called the SUSY-zero mechanism; when a U(1) symmetry is broken by vacuum expectation values (vev's) of positively [negatively] charged chiral multiplets, operators in the superpotential that appear to have negative [positive, respectively] charge are allowed because the vev's may supply the appropriate U(1) charge; but operators that appear to have positive [negative, resp.] U(1) charge—like (3) [(4), resp.] are not, because the multiplets with the vev's cannot supply negative [positive, resp.] U(1) charge so that the operators become U(1)-invariant. The vev's of the chiral multiplets $\bar{N}^c$ or $\bar{N}$ break the matter parity, yet the dimension-4 proton decay operators (2) are absent. Thus, the SUSY-zero mechanism can be an alternative to the matter parity.

The above argument, however, crucially depends on an assumption that the vev's are inserted only in non-negative power as in (3) and (4). In supersymmetric quantum field theories, in general, superpotential can be arbitrary, as long as it is holomorphic in chiral multiplets. It can also have a pole or singularity, on a Kähler manifold parametrized by chiral multiplets. Thus, it is hard to justify the absence of operators with $n < 0$ only with $\mathcal{N} = 1$ supersymmetry of D $= 4$ field theories. In effective field theories that arise from geometric compactification of string theory, however, we see in the following that we have a better answer to this question: the SUSY-zero mechanism is justified for renormalizable operators, though not necessarily for non-renormalizable operators. In particular, restricting to $n \geq 0$ in renormalizable operators (3) and (4) will be justified.

Heterotic $E_8 \times E_8$ string theory has a superpotential [3]

$$W \ni \int d^6 z \, \Omega \wedge \mathrm{tr}_{E_8\text{-adj.}} \left(A dA - i \frac{2}{3} AAA \right). \tag{5}$$

This describes a part of super Yang–Mills interactions. There are 16 supersymmetry charges locally, and these large supersymmetry and gauge symmetry constrain the superpotential to the form (5); note that it stops at the cubic term, and the α' corrections, vanish [4].[2] After compactification, this superpotential is re-written in terms of infinite number of D = 4 chiral multiplets. The superpotential (5) decomposes into supersymmetric mass terms and tri-linear interactions of chiral multiplets. Depending on which part of $\mathfrak{e}_8/\mathfrak{su}(5)_{\rm GUT}$ each chiral multiplet comes from, its representation under the $\mathrm{SU}(5)_{\rm GUT}\times\mathrm{U}(1)$ gauge group is different (see Table 1 and the appendix). We call them U(1) eigenstates. In terms of the U(1) eigenstates, the superpotential (5) consists of tri-linear interactions

$$\begin{aligned} W \ni\ & (y_u)_{ijk} \mathbf{10}_i . \mathbf{10}_j . H(\mathbf{5})_k + (y_{d,e})_{ijk} \bar{\mathbf{5}}_i \cdot \mathbf{10}_j \cdot \bar{H}(\bar{\mathbf{5}})_k + (y_\nu)_{ijk} \bar{N}_i \cdot \bar{\mathbf{5}}_j \cdot H(\mathbf{5})_k \\ + \ & (y^c_u)_{ijk} \mathbf{10}^c_i . \mathbf{10}^c_j . H(\mathbf{5})^c_k + (y^c_{d,e})_{ijk} \bar{\mathbf{5}}^c_i \cdot \mathbf{10}^c_j \cdot \bar{H}(\bar{\mathbf{5}})^c_k + (y^c_\nu)_{ijk} \bar{N}^c_i \cdot \bar{\mathbf{5}}^c_j \cdot H(\mathbf{5})^c_k \end{aligned} \tag{6}$$

and supersymmetric mass terms

$$\begin{aligned} W \ni\ & (M_{\bar{\mathbf{5}}})_{ij} \bar{\mathbf{5}}_i \cdot \bar{\mathbf{5}}^c_j + (M_{\bar{H}})_{ij} \bar{H}(\bar{\mathbf{5}})_i \cdot \bar{H}(\bar{\mathbf{5}})^c_j \\ & + (M_{\mathbf{10}})_{ij} \mathbf{10}_i \cdot \mathbf{10}^c_j + (M_{\bar{N}})_{ij} \bar{N}_i \cdot \bar{N}^c_j \ \left[+ (M_H)_{ij} H(\mathbf{5})_i \cdot H(\mathbf{5})^c_j \right] + \cdots . \end{aligned} \tag{7}$$

Indices i, j, k label infinite particles in the Kaluza–Klein tower of U(1) eigenstates. Note that $\bar{H}(\bar{\mathbf{5}})^c$-type U(1) eigenstates are nothing but the $H(\mathbf{5})$-type eigen states in the 4+1 model, but those two classes of states are different in the 3+2 model (see Table 1 and the appendix). The number of $\mathbf{10}$-type chiral multiplets should be larger than that of $\mathbf{10}^c$-type by 3, corresponding to the three generations of $(\bar{U}, Q, \bar{E})$. Similar chirality constraint exists for the multiplets in the $\mathrm{SU}(5)_{\rm GUT}$-**anti.-fund.** and **fund.** representations; String theory provides a dictionary translating topological information into this net chirality. Here, we just assume that the compactification geometry is chosen, so that the net chirality of the real world is reproduced. There is an additional constraint on the rank of the mass matrices, so that we have the electroweak Higgs doublets in the low-energy spectrum. The mass eigenvalues of the mass matrices $M_{\bar{\mathbf{5}}}$, $M_{\bar{H}}$ etc. are not necessarily either zero or of the order of the Kaluza–Klein scale; some mass eigenvalues are determined by moduli parameters of

[2]Although non-perturbative effects of string theory such as world-sheet instantons can generate extra contributions to the superpotential, we ignore them because their effects can be small.

compactification. There is no way specifying those eigenvalues without further specifying details of compactification, and we just leave them as arbitrary parameters of effective field theories. The terms of the superpotential (7) and (8), preserves the U(1) symmetry, as it should be.

As long as neither $\bar{N}$ nor $\bar{N}^c$ has non-zero expectation values, the U(1) symmetry is not broken. The distinction between the $\bar{\mathbf{5}}$-type and $\bar{H}(\bar{\mathbf{5}})$-type U(1) eigenstates is maintained, and the dimension-4 proton decay operator (2) is forbidden by the U(1) symmetry. This is the place we start off, and we examine how the expectation values of $\bar{N}$ or $\bar{N}^c$ would affect the low-energy effective field theories.

The superpotential of low-energy effective theory is obtained by i) identifying the massless modes, and ii) integrating out all but massless modes from the theory. Instead of dealing with infinite D = 4 chiral multiplets to be integrated out, we consider simpler models, where only finitely many Kaluza–Klein particles are maintained, so that we can deal with finite-by-finite mass matrices, instead of infinite-by-infinite ones. This is the approximation we are going to consider in the present paper.

When either $\bar{N}^c$ [or $\bar{N}$] develops a non-zero expectation value, the sixth term [or the third term] in (7) gives rise to a deformation in the supersymmetric mass matrix. Since a part of the mass matrices carry a non-zero U(1)-charge, mass eigenstates are no longer pure U(1) eigenstates. In particular, massless states in the SU(5)$_{\rm GUT}$-**anti.fund.** representation are no longer expected to be either pure $\bar{\mathbf{5}}$-type or pure $\bar{H}(\bar{\mathbf{5}})$-type U(1) eigenstates. Thus, this is potentially dangerous.

Let us first examine the massless modes in the simplified version of the 4+1 model, where only finitely many Kaluza–Klein particles are taken into account. Let us consider a model with

- 4 $\bar{\mathbf{5}}$-type chiral multiplets, denoted by $\bar{\mathbf{5}}_i$ $(i = 1, \cdots, 4)$,
- 1 $\bar{\mathbf{5}}^c$-type chiral multiplet,
- 2 $\bar{H}(\bar{\mathbf{5}})$-type chiral multiplets denoted by $\bar{H}_k$ $(k = 1, 2)$ and
- 2 $H(\mathbf{5})$-type chiral multiplets denoted by H_l $(l = 1, 2)$

(in addition to 3 **10**-type chiral multiplets). The supersymmetric mass matrix of multiplets in the SU(5)$_{\rm GUT}$-**fund.** and -**anti.fund.** representations is given by

$$W \ni (H_l, \bar{\mathbf{5}}^c) \begin{pmatrix} (M_H)_{lk} & 0 \\ (y^c_\nu \langle \bar{N}^c \rangle)_k & (M_{\bar{\mathbf{5}}})_i \end{pmatrix} \begin{pmatrix} \bar{H}_k \\ \bar{\mathbf{5}}_i \end{pmatrix}. \tag{8}$$

The 2 by 2 matrix $(M_H)_{lk}$ is assumed to be of rank 1, so that we have a pair of light Higgs doublets, H_u and H_d. Here, we have already chosen $\langle \bar{N}^c \rangle \neq 0$, and set $\langle \bar{N} \rangle = 0$, following [2]. Now, without a loss of generality, we can change the basis within H_l, $\bar{H}_k$ and $\bar{\mathbf{5}}_i$, so that $(M_H)_{lk} = 0$ except $(M_H)_{22} = M_H \neq 0$, and so that $(M_{\bar{\mathbf{5}}})_{i=1,2,3} = 0$ and $(M_{\bar{\mathbf{5}}})_4 = M_{\bar{\mathbf{5}}} \neq 0$. The 3 by 6 mass matrix (8) becomes

$$W \ni (H_1, H_2, \bar{\mathbf{5}}^c) \left(\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & M_H & 0 & 0 \\ (y_\nu^c \langle \bar{N}^c \rangle)_1 & (y_\nu^c \langle \bar{N}^c \rangle)_2 & 0 & M_{\bar{\mathbf{5}}} \end{array} \right) \left(\begin{array}{c} \bar{H}_1 \\ \bar{H}_2 \\ \bar{\mathbf{5}}_{1,2,3} \\ \bar{\mathbf{5}}_4 \end{array} \right). \tag{9}$$

We are primarily interested in the massless modes, which describe the low-energy effective theories. Although all the massive states are mixture of U(1) eigenstates, the U(1) eigenstates H_1 and $\bar{\mathbf{5}}_{1,2,3}$ are massless (and hence, mass eigen-) states, as well. Those low-energy multiplets, denoted by $\hat{}$ on it, namely $\hat{\bar{\mathbf{5}}}_{1,2,3} = \bar{\mathbf{5}}_{1,2,3}$ and $\hat{H} = H_1$ are identified with 3 generations of $(\bar{D}, L)$ and a quintet containing H_u, respectively. The other massless chiral multiplet, $\hat{\bar{\mathbf{5}}}_0$, is given by a linear combination $\propto \left(M_{\bar{\mathbf{5}}} \bar{H}_1 - (y_\nu^c \langle \bar{N}^c \rangle)_1 \bar{\mathbf{5}}_4 \right)$. This is to be identified with a quintet containing H_d. Thus, an important observation is that some of massless modes still remain to be U(1) eigenstates, although all the massive states are mixture of U(1) eigenstates.

It is important, in particular, that all three massless states $\hat{\bar{\mathbf{5}}}_{1,2,3} = (\bar{D}, L)_{1,2,3}$ remain to be pure $\bar{\mathbf{5}}$-type U(1) eigenstates. When the superpotential (7) written in terms of U(1) eigenstates are re-written in terms of mass eigenstates, and when terms that only involve massless states are retained, the U(1) eigenstates $\bar{\mathbf{5}}_i$ turn into both $\hat{\bar{\mathbf{5}}}_{1,2,3} = (\bar{D}, L)_{1,2,3}$ and $\hat{\bar{\mathbf{5}}}_0 = H_d$, but $\bar{H}_k$ turn only into $\hat{\bar{\mathbf{5}}}_0 = H_d$, not to $\hat{\bar{\mathbf{5}}}_{1,2,3}$. Thus, the dimension-4 operator (2) that involves two of $\hat{\bar{\mathbf{5}}}_{1,2,3} = (\bar{D}, L)_{1,2,3}$ does not arise from the super Yang–Mills interaction (7) and (8), even when $\langle \bar{N}^c \rangle \neq 0$ and the matter parity is broken. This result has been obtained in [2], although phrased in a more geometric language (see also the appendix). Note also that a lepton number violating operator

$$W \ni y_{d,e} \langle \bar{N}^c \rangle H_d \cdot \bar{E} \cdot H_d \tag{10}$$

could have been generated, but it vanishes when there is only on doublet because of anti-symmetric contraction of SU(2) indices.

Let us continue the analysis a little further, before drawing a conclusion on the dimension-4 proton decay operator. The same analysis can be carried out,[3] now assuming $\langle \bar{N} \rangle \neq 0$ and

[3]There, one has to assume that the 2 by 6 matrix $((M_H)_{lk}, (y_\nu \langle \bar{N} \rangle)_{li})$ is of rank 1.

$\langle \bar{N}^c \rangle = 0$, instead of the other way around. This case should lead to dimension-4 proton decay [2], and let us confirm it in the D = 4 field-theory language we used above. We can see by diagonalizing the mass matrix that massive states are mixture of U(1) eigenstates, but some of massless states remain to be pure U(1) eigenstates, just as in the previous case with $\langle \bar{N}^c \rangle \neq 0, \langle \bar{N} \rangle = 0$. The massless states $\hat{H} = H_1 \supset H_u$ and $\hat{\bar{\mathbf{5}}}_{1,2} = (\bar{D}, L)_{1,2}$ still remain pure U(1) eigenstates, just as in the previous case. The difference from the previous case is that $\hat{\bar{H}} = \bar{H}_1 \supset H_d$ becomes a pure $\bar{H}(\bar{\mathbf{5}})$-type U(1) eigenstate, and the other massless state $\hat{\bar{\mathbf{5}}}_3 = (\bar{D}, L)_3 \propto \left(M_H \bar{\mathbf{5}}_3 - (y_\nu \langle \bar{N} \rangle)_3 \bar{H}_2\right)$ becomes the mixture of U(1) eigenstates, instead. When converting the superpotential (7)+(8) in the U(1) eigenbasis into the mass eigenbasis and retaining only the massless states, $\bar{H}_2$ contains the $\hat{\bar{\mathbf{5}}}_3$ component, and the second term of (7) becomes

$$y_{d,e}\, \bar{\mathbf{5}}_{1,2,3} \cdot \mathbf{10} \cdot \bar{H}_{1,2} \to y_{d,e}\, \hat{\bar{\mathbf{5}}}_{1,2} \cdot \mathbf{10} \cdot H_d + y_{d,e}\, \hat{\bar{\mathbf{5}}}_3 \cdot \mathbf{10} \cdot H_d - y_{d,e}\, \hat{\bar{\mathbf{5}}}_{1,2} \cdot \mathbf{10} \cdot \left(\frac{y_\nu \langle \bar{N} \rangle}{\sqrt{|M_H|^2 + |(y_\nu \langle \bar{N} \rangle)|^2}} \right) \hat{\bar{\mathbf{5}}}_3. \tag{11}$$

Thus, the dimension-4 proton decay operator is indeed generated when $\langle \bar{N} \rangle \neq 0$. The $\langle \bar{N} \rangle$-dependence of the last term explains why the single $\langle \bar{N} \rangle$ insertion captures the physics at the level of whether certain operators vanish or not, even when $\langle \bar{N} \rangle \gg M_H$. The single $\langle \bar{N} \rangle$ insertion was also used in the discussion in [2], where it was backed by a geometric intuition. Now we have an independent confirmation of how and why it works.

So far, we have discussed only renormalizable (dimension-4) operators in the low-energy effective theories. The renormalizable part of the low-energy effective superpotential is given by re-writing the U(1) eigenstates in terms of mass eigenstates and just drop all the terms containing heavy states [4]. We have seen in both cases, namely $\langle \bar{N}^c \rangle \neq 0$ and $\langle \bar{N} \rangle \neq 0$, in the D = 4 field theory language (and in a more geometric language in [2]) that the mixing between the U(1) eigenstates can be traced in the low-energy effective superpotential by single ($n = 1$) insertion of the expectation values $\langle \bar{N}^c \rangle$ and $\langle \bar{N} \rangle$. As a consequence, we have seen that $\langle \bar{N} \rangle \neq 0$ does generate dimension-4 proton decay operator (2), but the low-energy effective theories remain free of (2) as long as $\langle \bar{N} \rangle = 0$, even if $\langle \bar{N}^c \rangle \neq 0$ and the matter parity is not preserved. Similar analyses can be carried out for the 3+2 model, but it is essentially the same as in the 4+1 model, and we do not repeat here.

We have so far implicitly assumed that the $\bar{\mathbf{5}}$–$\bar{H}(\bar{\mathbf{5}})$ mixing arises only from the superpotential. This is the case if the Kähler potential is of the form

$$
\begin{aligned}
K &= Z^{\bar{\mathbf{5}}}_{ij}\bar{\mathbf{5}}^\dagger_i\bar{\mathbf{5}}_j + Z^{\bar{H}}_{ij}\bar{H}^\dagger_j\bar{H}_j && (12)\\
&+ c_{ijkl}\bar{\mathbf{5}}^\dagger_i\bar{\mathbf{5}}^\dagger_j\bar{\mathbf{5}}_k\bar{\mathbf{5}}_l + c'_{ijkl}\bar{H}^\dagger_i\bar{H}^\dagger_j\bar{H}_k\bar{H}_l + c''_{ijkl}\bar{\mathbf{5}}^\dagger_i\bar{\mathbf{5}}_j\bar{N}^\dagger_k\bar{N}_l + \cdots . && (13)
\end{aligned}
$$

However, $\mathrm{U}(5)_{\mathrm{GUT}} \times \mathrm{U}(1)$ invariant Kähler potential of $\mathcal{N} = 1$ supersymmetry may have such terms as

$$
K \ni \kappa_n \bar{\mathbf{5}}^\dagger \bar{N}^c \bar{H}(\bar{N}^{c\dagger}\bar{N}^c)^n + \lambda_n \bar{\mathbf{5}}^\dagger \bar{N}^\dagger \bar{H}(\bar{N}^\dagger \bar{N})^n + \text{h.c.}, \tag{14}
$$

which leads to kinetic $\bar{\mathbf{5}}$–$\bar{H}$ mixing when either $\langle \bar{N}^c \rangle$ or $\langle \bar{N} \rangle$ is non-zero. Even if the Kähler potential contains those terms, it turns out that they are not a problem. We can re-define the chiral multiplets as

$$
\bar{\mathbf{5}}' = \bar{\mathbf{5}} + (Z^{\bar{\mathbf{5}}})^{-1}\left(\kappa_n \langle \bar{N}^c \rangle |\langle \bar{N}^c \rangle|^{2n}\bar{H} + \lambda_n \langle \bar{N} \rangle^* |\langle \bar{N} \rangle|^{2n}\bar{H}\right), \tag{15}
$$

so that the bi-linear part of the Kähler potential is like (13) in the newly defined chiral multiplets. The superpotential should also be re-written at the same time; $\bar{\mathbf{5}}$ in the superpotential is replaced by $\bar{\mathbf{5}}' - Z^{-1}(\kappa_0 \langle \bar{N}^c \rangle + \lambda_0 \langle \bar{N} \rangle + \cdots)\bar{H}$. Thus, operators of the form $W \ni \bar{H}\cdot\mathbf{10}\cdot\bar{H}$ are generated, but as long as we have only one chiral multiplet that have the same properties as that of H_d in the low-energy spectrum, this operator vanishes because it is anti-symmetric under the exchange of two $\bar{H}$'s. Thus, the whole arguments for the absence of dimension-4 proton decay operator (2) are not affected, even when the Kähler potential has terms like (14).

Now that we have seen that the framework proposed in [2] guarantees that the dangerous dimension-4 operator (2) is absent, let us move on to discuss non-renormalizable operators. Renormalizable (dimension-4) terms of the effective superpotential is obtained by truncating all the terms that involve heavy states, but the heavy states should not be just truncated, but should be integrated out to obtain the superpotential of the effective theories. Lots of non-renormalizable operators are generated, in general, when heavy states are integrated out.[4]

Instead of trying to be general, let us look at explicit examples. Figure 1 (a) is a super Feynman diagram, showing that

$$
W \ni \frac{y_u y_{d,e} y^c_\nu \langle \bar{N}^c \rangle}{M_{\bar{\mathbf{5}}} M_H}\, \mathbf{10}\cdot\mathbf{10}\cdot\mathbf{10}\cdot\bar{H}(\bar{\mathbf{5}}) \tag{16}
$$

[4]Renormalizable terms are not affected by this process [4].

is generated in the 4+1 model, if there are massive vector-like pairs $(H_2, \bar{H}_2)$ and $(\bar{\mathbf{5}}^c, \bar{\mathbf{5}}_4)$. One can also check that this operator is neutral under the U(1)-charge assignment of the 4+1 model in Table 1. $M_H,\ M_{\bar{\mathbf{5}}} \gg \langle \bar{N}^c \rangle$ is assumed in the coefficient. Another example is Figure 1 (b), where we see that

$$W \ni \frac{y_u y_{d,e}}{y_\nu \langle \bar{N} \rangle}\, \mathbf{10} \cdot \mathbf{10} \cdot \mathbf{10} \cdot \bar{H}(\bar{\mathbf{5}}) \tag{17}$$

can be generated in the 3+2 model, if there is an extra pair of $\bar{\mathbf{5}}$-type and $H(\mathbf{5})$-type chiral multiplet. This operator is also neutral under the U(1)-charge assignment of the 3+2 model in Table 1. It is worth noting that the expectation value breaking the anomalous U(1) gauge symmetry can appear not only in the numerator but also sometimes in the denominator. In the latter example, the vector-like pair of chiral multiplets have a mass term only through the Dirac neutrino Yukawa coupling involving the expectation value of $\bar{N}$. When the expectation value vanishes, the vector-like pair is massless, and the number of massless modes changes. The locus of $\langle \bar{N} \rangle = 0$ is a singular locus of the moduli space in the topological sector. In such situation, the expectation value breaking the U(1) symmetry appear in the denominator of coefficients of non-renormalizable operators in the effective theories. The negatively charged $\langle \bar{N} \rangle$ in the denominator supplies positive U(1) charges in (17), neutralizing the negative charge of $\mathbf{10} \cdot \mathbf{10} \cdot \mathbf{10} \cdot \bar{H}$. Thus, this operator is not eliminated. This example clearly shows that the SUSY-zero mechanism does not necessarily work for the non-renormalizable operators.

The dimension-5 operators above effectively look like

$$W_{\text{eff.}} \ni \frac{1}{M_{\text{eff.}}} \mathbf{10} \cdot \mathbf{10} \cdot \mathbf{10} \cdot \bar{H}(\bar{\mathbf{5}}) \tag{18}$$

in low-energy physics, whether it comes from the 4+1 model or from the 3+2 model. Further non-renormalizable operators are generated by integrating out heavy states, and the effective superpotential does not necessarily stop at finite-degree polynomial. This operator, however, can be a leading contribution to the new physics beyond the supersymmetric standard model, and we work on this operator in the rest of this article. It is written in terms of chiral multiplets of the minimal supersymmetric standard model.as

$$W_{\text{eff.}} \ni \frac{1}{M_{\text{eff.}}} Q\ Q\ Q\ H_d + \frac{1}{M'_{\text{eff.}}} Q\ \bar{U}\ \bar{E}\ H_d \tag{19}$$

Baryon number, lepton number symmetries and the matter parity are broken by these dimension-5 operators. The effective coefficients of the two operators are not necessarily exactly the same, because of various $\mathrm{SU}(5)_{\mathrm{GUT}}$ symmetry breaking effects, such as the Wilson line.[5] The effective energy scale $M_{\mathrm{eff.}}$ and $M'_{\mathrm{eff.}}$ depend on the tri-linear couplings in (7), the expectation value $\bar{N}$ or $\bar{N}^c$, and on expectation values of $\mathrm{SU}(5)_{\mathrm{GUT}}$-singlet moduli fields that affect the mass eigenvalues such as M_H and $M_{\bar{\mathbf{5}}}$. Although one could come up with some naive order-of-magnitude estimate of the first two,[6] the stabilization of vector bundle moduli is poorly understood in the current string theory, and string theory is not able to make a unique prediction of the effective energy scale.

Instead, we constrain the range of those energy scales by phenomenological limits. The first operator breaks baryon number, and the latter lepton number. Since proton decay process has to involve both baryon number violation and lepton number violation, proton decay amplitudes from (18) should involve both operators of (19). The lifetime is proportional to $(M_{\mathrm{eff.}}M'_{\mathrm{eff}})^2$. Thus, the operators such as (19) are consistent with the proton decay experiments as long as the geometric mean of $M_{\mathrm{eff.}}$ and $M'_{\mathrm{eff.}}$ are large enough, say of order of the GUT scale or lager.

The operators (19) also lead to LSP decay. Either one of those operators is enough. Figure 2 are some of Feynman diagrams contributing to the LSP decay when the LSP is a neutralino. Sleptons or squarks also decay e.g. through the Feynman diagrams in Fig. 3), if they are the LSP. Since the limit from the proton decay only constrains the product of $M_{\mathrm{eff.}}$ and $M'_{\mathrm{eff.}}$, the decay rate of slepton LSP is not constrained at all. On the other hand, both operators in (19) contribute to the decay amplitude in the case of neutralino or squark LSP, and the decay amplitude may be dominated by amplitudes involving only either one of them.

When the both effective energy scales $M_{\mathrm{eff.}}$ and M'_{eff} are roughly of the same order, the LSP lifetime is approximately

$$\tau \approx \frac{M^2_{\mathrm{eff.}}}{(100 \text{ GeV})^3} \approx 1 \text{ min.} \left(\frac{M_{\mathrm{eff.}}}{10^{16} \text{ GeV}}\right)^2, \tag{20}$$

showing that the LSP decays roughly at the epoch of the big-bang nucleosynthesis if $M_{\mathrm{eff.}}$ is of order 10^{16} GeV. To be more precise, $M_{\mathrm{eff.}}$ and $M'_{\mathrm{eff.}}$ have generation indices, and those that matter to proton decay and the LSP decay are not the same. Since the LSP decay process picks up the largest one, while proton decay does not necessarily, M_{eff} that determines the

[5] Although (7) is written in an $\mathrm{SU}(5)_{\mathrm{GUT}}$-symmetric way, it was just for brevity of notation. The spectra and wave functions of mass eigenstates should be different for $\mathrm{SU}(5)_{\mathrm{GUT}}$ partners, due to the $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking effects, such as the Wilson line, and hence the coefficients in (7) are not actually $\mathrm{SU}(5)_{\mathrm{GUT}}$-symmetric. Thus, $M_{\mathrm{eff.}}$ and M'_{eff} are not expected to be exactly the same, either.

[6] One could use the order of magnitude of the Yukawa couplings we already know for the tri-linear Yukawa couplings; for the naive order-of-magnitude estimate for $\langle \bar{N}^c \rangle$ or $\langle \bar{N} \rangle$, see [2].

LSP lifetime may be even lower than 10^{16} GeV, meaning that the LSP may decay even before the big-bang nucleosynthesis. But, the precise lifetime further depends on the mixing among neutralinos, for instance, in the case of neutralino LSP, and requires detailed calculations. If the LSP decays after the big-bang nucleosynthesis, the relic abundance of the LSP (before the decay) is constrained, and so is the thermal history of the universe, consequently.

If the lifetime is short enough, say, $c\tau\gamma \lesssim 10$km, some of supersymmetry particles produced at the LHC decay inside the detectors. The LSP that decays does not contribute to the missing energy. Jets (and possibly a lepton) come out of a displaced vertex in LSP decay events. If the lifetime is not that short, the LSP decay outside the detectors and we may not notice. But, in the case the LSP is a charged particle, say a slepton or a squark, they can be trapped; such experiments have been proposed in the context of the NLSP decay to the gravitino in the gravitino LSP scenario [5]. The LSP decay events are completely different from the NLSP decay to gravitino, and it would not be difficult to make a distinction between them. It would be further interesting if the branching ratios of various decay modes of the LSP can be measured, since both $M_{\text{eff.}}$ and $M'_{\text{eff.}}$ can be extracted. We already know that the Yukawa couplings of strange quark and muon do not really unify. Thus, the measurement of the two energy scales, $M_{\text{eff.}}$ and $M'_{\text{eff.}}$, may give us another clue to understand how the $\text{SU}(5)_{\text{GUT}}$ unified symmetry is broken. Further detailed phenomenological study will be presented elsewhere [6].

If the LSP lifetime is shorter than the current age of the universe, it cannot be a candidate of dark matter. This makes axion an attractive candidate of dark matter. The Peccei–Quinn mechanism still remains one of the best solutions to the strong CP problem, which predicts an axion field. The relic abundance of the axion may be explained by anthropic choice of the initial amplitude of the axion filed [7]. It should be reminded that even if the LSP that does not decay within the detectors of the LHC, it is still not necessarily stable in the cosmological timescale. Indeed, we have seen that it may really be the case in a theoretically well-motivated framework.

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Appendix

In Heterotic string theory with $E_8 \times E_8$ gauge group, a rank-5 vector bundle V_5 has to be turned on in one of E_8 in order to obtain an $\mathrm{SU}(5)_{\mathrm{GUT}}$ unified theory. For the 4+1 and 3+2 models of [2] the V_5 is taken at reducible limits such as $V_5 = U_4 \oplus L$, where U_4 is rank-4 bundle and L a line bundle, or $V_5 = U_3 \oplus U_2$, where U_3 and U_2 are rank-3 and -2 vector bundles, respectively. The structure group of the rank-5 bundle is reduced from SU(5) to either SU(4) $\times$ $\mathrm{U}(1)_\chi$ or SU(3) $\times$ SU(2) $\times$ $\mathrm{U}(1)_{\tilde{q}_7}$. The commutant of the structure group in E_8 is either $\mathrm{SU}(5)_{\mathrm{GUT}} \times \mathrm{U}(1)_\chi$ or $\mathrm{SU}(5)_{\mathrm{GUT}} \times \mathrm{U}(1)_{\tilde{q}_7}$, the gauge group discussed in the main text of this article.

In the 4+1 model, U_4-valued (0,1)-form become (scalar part of) **10**-type chiral multiplets, and $\overline{U}_4$-valued (0,1)-form become $\mathbf{10}^c$-type chiral multiplets. The $\mathrm{U}(1)_\chi$ charges of **10**-type chiral multiplets are -1, as shown in Table 1. The massless modes are $H^1(Z; U_4)$ and $H^1(Z; \overline{U}_4)$, where Z is a Calabi–Yau 3-fold for compactification. The $\bar{\mathbf{5}}$-type, $\bar{H}(\bar{\mathbf{5}}) = H(\mathbf{5})^c$-type and $\bar{N}$-type chiral multiplets are from $U_4 \otimes L$-, $\wedge^2 U_4$- and $U_4 \otimes L^{-1}$-valued (0,1)-forms, respectively, and the $\bar{\mathbf{5}}^c$-type, $H(\mathbf{5})$-type and $\bar{N}^c$-type chiral multiplets are from bundles in the Hermitian conjugate representation, $\overline{U_4 \otimes L}$, $\overline{\wedge^2 U_4}$ and $\overline{U}_4 \otimes L$. Note that $\bar{N}^c$-type chiral multiplets were denoted as $\bar{\bar{N}}$ in [2].

In the 3+2 model, U_2-valued (0,1)-form become **10**-type chiral multiplets, and $\mathbf{10}^c$-type multiplets are from the $\overline{U_2}$ bundle. The $\bar{\mathbf{5}}$-type, $\bar{H}(\bar{\mathbf{5}})$-type and $H(\mathbf{5})^c$-type chiral multiplets originate from the $U_3 \otimes U_2$-, $\wedge^2 U_3$- and $\wedge^2 U_2$ bundle valued (0,1)-form, and $\bar{\mathbf{5}}^c$-type, $\bar{H}(\bar{\mathbf{5}})^c$-type and $H(\mathbf{5})$-type chiral multiplets are from their Hermitian conjugate bundles. $\bar{N}$-type multiplets are from $\overline{U_3} \otimes U_2$, and $\bar{N}^c$-type from $U_3 \otimes \overline{U_2}$. The massless modes are given by the first cohomology of the corresponding vector bundles. The $\mathrm{U}(1)_{\tilde{q}_7}$ charges are shown in Table 1.

The difference between the $\bar{\mathbf{5}}$-type and $\bar{H}(\bar{\mathbf{5}})$-type chiral multiplets is not only due to their different U(1) charges. Although we discussed the selection rule that follows only from the U(1) symmetry in this article, [2] discusses the selection rules that come from the underlying gauge symmetry SU(4) or SU(3)$\times$SU(2) as well. The centre of the structure group, $\mathbb{Z}_4$ and $\mathbb{Z}_3 \times \mathbb{Z}_2$, respectively, may not be broken by the gauge connection describing a vector bundle V_5. In the 4+1 model, the expectation value of $\bar{N}^c$-type chiral multiplets leave the diagonal subgroup of the matter parity $\mathbb{Z}_{10}/\mathbb{Z}_5 \simeq \mathbb{Z}_2$ and the $\mathbb{Z}_2$ subgroup of the centre $\mathbb{Z}_4$ unbroken. But, it turns out that all the chiral multiplets are even under this diagonal unbroken $\mathbb{Z}_2$ symmetry, and this unbroken symmetry does not have any significance. In the 3+2 model, the diagonal subgroup of the matter parity and the centre of the structure group SU(2) may remain unbroken, but all the chiral multiplets are even under this diagonal $\mathbb{Z}_2$ symmetry. Thus, it does not lead to a selection rule that is not covered in this article.

Reference [2] only discussed the wave functions of the zero modes in its section 3, because we were interested only in the renormalizable part of the effective theory. In this article, we started out with field-theory models that contain not only the zero modes but also Kaluza–Klein modes and vector-like pair of zero modes that do not contribute to the net chirality. In the end, however, what we did is to determine the massless modes (zero modes) by diagonalizing mass matrices. Thus, the analysis in the former half of this article is essentially the same as what we did in section 3 of [2]. The difference is that we used differential equation in internal space and geometric intuition in [2], while we used truncated $D = 4$ spectra and diagonalization of mass matrices in this article.

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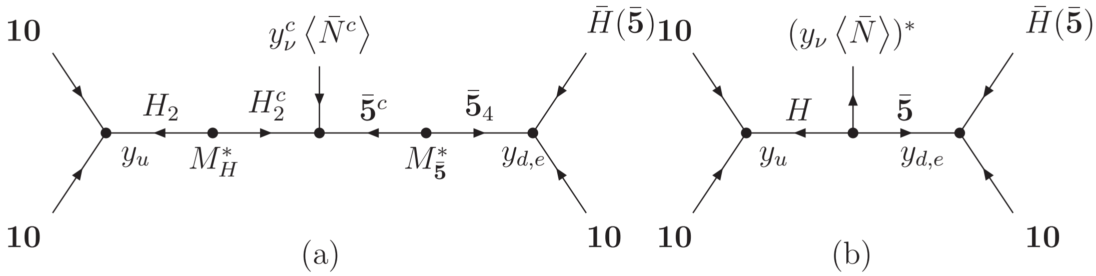

Figure 1: Super Feynman diagrams for the LSP decay operators.

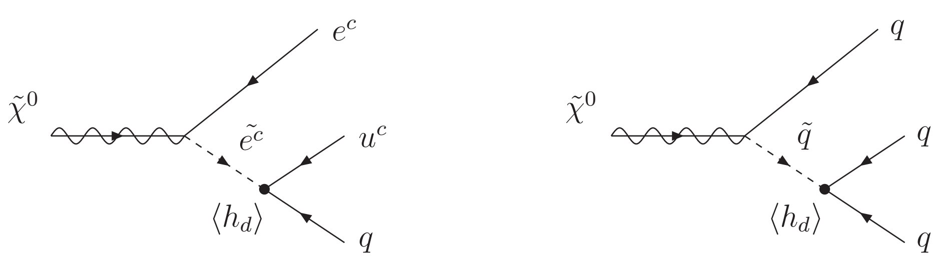

Figure 2: Feymann diagrams of neutralino decay.

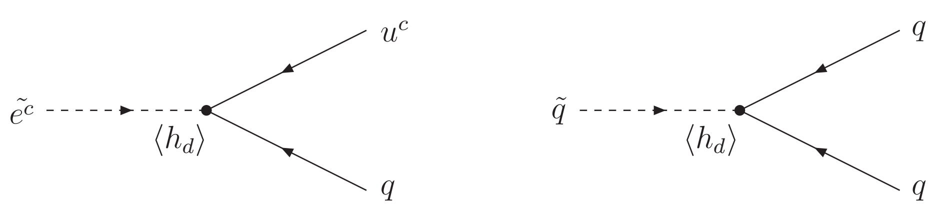

Figure 3: Feymann diagrams of slepton and squark decay.